## 7.2 - Properties of Chords in a Circle

A line segment that joins two points on a circle is called a chord.
A chord that goes through the centre of a circle is called the diameter.


## Chord Properties

Property 1: The perpendicular from the centre of a circle to a chord bisects the chord. This means that the perpendicular divides the
 chord into two equal parts.

Property 2: The perpendicular bisector of a chord in a circle passes through the centre of the circle.


Property 3: A line that joins the centre of a circle and the midpoint of a chord is perpendicular to the chord.


Ex. 1 Point O is the centre of the circle, and the line segment OC bisects chord $\mathrm{AB} . \angle \mathrm{OAC}=33^{\circ}$. Determine the values of $x^{\circ}$ and $y^{\circ}$.

Answer: $\quad$ Since $A C=B C, O C$ bisects the chord $A B$.
Therefore, $O C$ must be perpendicular to $A B$, and $\angle \mathrm{ACO}=90^{\circ}$.
Since OA and OB are both radii, they are equal, which makes this an isosceles triangle. Therefore, $\angle \mathrm{OAB}=\angle \mathrm{OBC}=33^{\circ}$.
Therefore, $x^{\circ}=33^{0}$.


To find $y^{\circ}$, all angles in $\triangle A O C$ add up to $180^{\circ}$.
$180-90-33=57^{\circ}$
Therefore, $\boldsymbol{y}^{\circ}=33^{\circ}$

Ex. 2 Point O is the centre of a circle.
$A B$ is a diameter with length 26 cm .
CD is a chord that is 10 cm from the centre of the circle. What is the length of chord CD?

Answer: $\quad \mathrm{AB}$ is the diameter, so AO is the radius, which will be equal to $O C$. $A O=O C=13 \mathrm{~cm}$.


Use Pythagorean's Theorem to find CE.

$$
\begin{aligned}
& x^{2}+10^{2}=13^{2} \\
& x^{2}=169-100 \\
& x^{2}=69 \\
& x=\sqrt{69}=8.3 \mathrm{~cm}
\end{aligned}
$$

$$
C E=E D \text { so } C D=2(8.3)=16.6 \mathrm{~cm}
$$

Ex. 3 A horizontal pipe has a circular cross section, with centre O . Its radius is 20 cm . Water fills less than one-half of the pipe. The surface of the water $A B$ is 24 cm wide. Determine the maximum depth of the water, which is the depth CD.

Answer:

$$
C D=O D-O C
$$

Since $O C$ is perpendicular to $A B$, it cuts
 $A B$ in half, therefore, $A C=1 / 2(24)=12 \mathrm{~cm}$

Use Pythagorean's Theorem to find OC:
$\mathrm{x}^{2}+12^{2}=20^{2}$
$\mathrm{x}^{2}=400-144$
$\mathrm{x}^{2}=256$
$x=\sqrt{256}=16 \mathrm{~cm}$
$20 \mathrm{~cm}-16 \mathrm{~cm}=4 \mathrm{~cm}$
Therefore, the water is $\mathbf{4} \mathbf{~ c m}$ deep.

