Exponent Laws II

Reflect & Share

What patterns do you see in the rows of the table?
Compare your patterns with those of another pair of classmates.
Use these patterns to record a rule for:

- writing the power of a power as a single power
- writing the power of a product as a product of two powers

How can you check your rules?

Connect

We can use the exponent laws from Lesson 2.4 to simplify powers written in other forms.

* ➤ Power of a power

We can raise a power to a power.
For example, 3^2 raised to the power 4 is written as $(3^2)^4$.
$(3^2)^4$ is a *power of a power.*
$(3^2)^4$ means $3^2 \times 3^2 \times 3^2 \times 3^2$.
So, $3^2 \times 3^2 \times 3^2 \times 3^2 = 3^{2+2+2+2}$ Using the exponent law for the product of powers
$= 3^8$
The exponent of 3^8 is the product of the exponents in $(3^2)^4$.
That is, $(3^2)^4 = 3^{2 \times 4}$
$= 3^8$
We can use this result to write an exponent law for the power of a power.

▸ Exponent Law for a Power of a Power

To raise a power to a power, multiply the exponents.
$(a^m)^n = a^{mn}$ — mn means $m \times n$
a is any integer, except 0.
m and n are any whole numbers.

➤ Power of a product

The base of a power may be a product; for example, $(3 \times 4)^5$.
$(3 \times 4)^5$ is a *power of a product.*
$(3 \times 4)^5$ means $(3 \times 4) \times (3 \times 4) \times (3 \times 4) \times (3 \times 4) \times (3 \times 4)$
So, $(3 \times 4) \times (3 \times 4) \times (3 \times 4) \times (3 \times 4) \times (3 \times 4)$
$= 3 \times 4 \times 3 \times 4 \times 3 \times 4 \times 3 \times 4 \times 3 \times 4$ Removing the brackets
$= (3 \times 3 \times 3 \times 3 \times 3) \times (4 \times 4 \times 4 \times 4 \times 4)$ Grouping equal factors
$= 3^5 \times 4^5$ Writing repeated multiplications as powers

We can use this result to write an exponent law for the power of a product.

- **Exponent Law for a Power of a Product**
  $(ab)^m = a^m b^m$
  a and b are any integers, except 0.
  m is any whole number.

➤ Power of a quotient

The base of a power may be a quotient; for example, $\left(\frac{5}{6}\right)^3$.

$\left(\frac{5}{6}\right)^3$ **is a power of a quotient.**

$\left(\frac{5}{6}\right)^3$ means $\left(\frac{5}{6}\right) \times \left(\frac{5}{6}\right) \times \left(\frac{5}{6}\right)$

So, $\left(\frac{5}{6}\right) \times \left(\frac{5}{6}\right) \times \left(\frac{5}{6}\right) = \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6}$

$= \frac{5 \times 5 \times 5}{6 \times 6 \times 6}$ Multiplying the fractions

$= \frac{5^3}{6^3}$ Writing repeated multiplications as powers

We can use this result to write an exponent law for the power of a quotient.

- **Exponent Law for a Power of a Quotient**
  $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ $b \neq 0$
  a and b are any integers, except 0.
  n is any whole number.

We can use these exponent laws to simplify or evaluate an expression.

Example 1 Simplifying a Power of a Power

Write as a power.

a) $[(-7)^3]^2$ b) $-(2^4)^5$ c) $(6^2)^7$

➤ A Solution

Use the exponent law for a power of a power.

a) $[(-7)^3]^2 = (-7)^{3 \times 2}$
$= (-7)^6$

b) $-(2^4)^5 = -(2^{4 \times 5})$
$= -2^{20}$

c) $(6^2)^7 = 6^{2 \times 7}$
$= 6^{14}$

Example 2 Evaluating Powers of Products and Quotients

Evaluate.

a) $[(-7) \times 5]^2$
b) $[24 \div (-6)]^4$
c) $-(3 \times 2)^2$
d) $\left(\frac{78}{13}\right)^3$

Solutions

Method 1

a) Use the exponent law for a power of a product.

$$[(-7) \times 5]^2 = (-7)^2 \times 5^2$$
$$= 49 \times 25$$
$$= 1225$$

b) Use the exponent law for a power of a quotient. Write the quotient in fraction form.

$$[24 \div (-6)]^4 = \left(\frac{24}{-6}\right)^4$$
$$= \frac{24^4}{(-6)^4}$$
$$= \frac{331\,776}{1296}$$
$$= 256$$

c) Use the exponent law for a power of a product.

$$-(3 \times 2)^2 = -(3^2 \times 2^2)$$
$$= -(9 \times 4)$$
$$= -36$$

d) Use the exponent law for a power of a quotient.

$$\left(\frac{78}{13}\right)^3 = \frac{78^3}{13^3}$$
$$= \frac{474\,552}{2197}$$
$$= 216$$

Method 2

Use the order of operations.

a) $[(-7) \times 5]^2 = (-35)^2$
$= 1225$

b) $[24 \div (-6)]^4 = (-4)^4$
$= 256$

c) $-(3 \times 2)^2 = -(6)^2$
$= -6^2$
$= -36$

d) $\left(\frac{78}{13}\right)^3 = 6^3$
$= 216$

Example 3 Applying Exponent Laws and Order of Operations

Simplify, then evaluate each expression.

a) $(3^2 \times 3^3)^3 - (4^3 \times 4^2)^2$ b) $(6 \times 7)^2 + (3^8 \div 3^6)^3$ c) $[(-5)^3 + (-5)^4]^0$

A Solution

Use the exponent laws to simplify first, where appropriate.

a) In each set of brackets, the bases are the same, so use the exponent law for products.

$(3^2 \times 3^3)^3 - (4^3 \times 4^2)^2$

$= (3^{2+3})^3 - (4^{3+2})^2$ Add the exponents in each set of brackets.

$= (3^5)^3 - (4^5)^2$ Use the power of a power law.

$= 3^{5 \times 3} - 4^{5 \times 2}$ Multiply the exponents.

$= 3^{15} - 4^{10}$ Use a calculator.

$= 14\,348\,907 - 1\,048\,576$

$= 13\,300\,331$

b) Multiply in the first set of brackets. Use the exponent law for the quotient of powers in the second set of brackets.

$(6 \times 7)^2 + (3^8 \div 3^6)^3$

$= (42)^2 + (3^{8-6})^3$

$= 42^2 + (3^2)^3$ Use the power of a power law.

$= 42^2 + 3^6$ Use a calculator.

$= 1764 + 729$

$= 2493$

c) The expression is a power with exponent 0, so its value is 1.

$[(-5)^3 + (-5)^4]^0 = 1$

Discuss the Ideas

1. Why do you add the exponents to simplify $3^2 \times 3^4$, but multiply the exponents to simplify the expression $(3^2)^4$?

2. a) What is the difference between a quotient of powers and a power of a quotient?
   b) What is the difference between a product of powers and a power of a product?

3. In *Example 3*, is it easier to key the original expressions in a calculator or use the exponent laws to simplify first? Justify your answer.