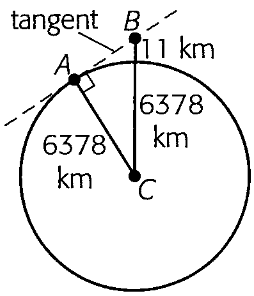
EXAMPLE 3 Using tangents to solve problems

An aircraft is at an altitude of 11 km. Earth has an average radius of about 6378 km. How far from the plane is the horizon, to the nearest kilometre?

Nola's Solution



First, I drew a sketch of the problem.
I drew a tangent from the plane to Earth's surface at point A (the horizon).

I knew that A is a point of tangency because the line of sight from the plane to the horizon touches the Earth only at point A.

I drew Earth's radius and formed a right triangle.

$$BC = 11 + 6378$$
$$= 6389 \text{ km}$$

I calculated the length of BC by adding the altitude to Earth's radius.

$$AB^2 + AC^2 = BC^2$$
$$AB^2 = BC^2 - AC^2$$
$$AB^2 = 6389^2 - 6378^2$$
$$= 140\,437$$
$$AB \doteq 375 \text{ km}$$

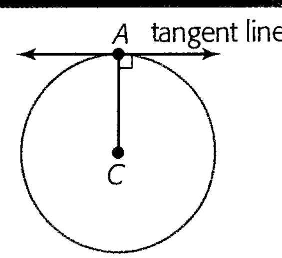
I used the Pythagorean theorem to calculate the distance to the horizon.

The horizon is about 375 km from the plane.

In Summary

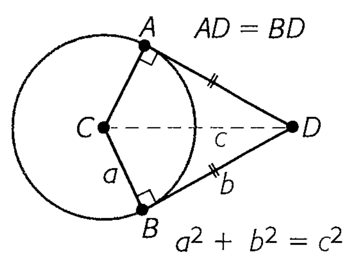
Key Idea

- A tangent to a circle is perpendicular to the radius drawn to the point of tangency.



Need to Know

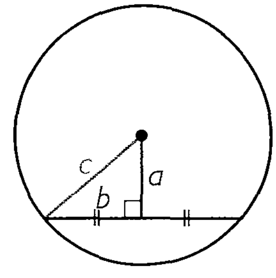
- Tangent segments drawn from an external point to a circle are equal.
- Tangent properties and the Pythagorean theorem can be used to solve circle problems.



In Summary

Key Idea

- Since a line from the centre of a circle to the midpoint of a chord is the perpendicular bisector of the chord, the Pythagorean theorem can be used to calculate how far the chord is from the centre of the circle.



$a^2 + b^2 = c^2$

Need to Know

- Chords equidistant from the centre of a circle are of equal length.
- Chords of equal length are equidistant from the centre of a circle.

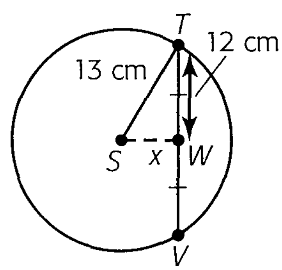
Communication *Tip*

To describe the distance between the centre of a circle and a chord, use the measured distance from the centre of the circle to the midpoint of the chord.

Checking

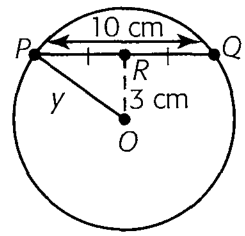
1. Calculate the missing lengths to the nearest centimetre.

a)



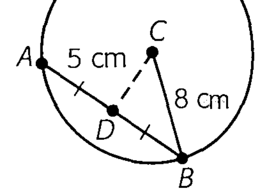
$x = ?$

b)



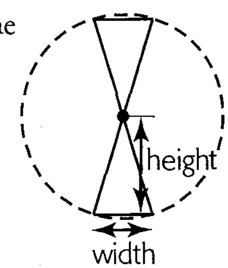
$y = ?$

c)



$AB = ?$

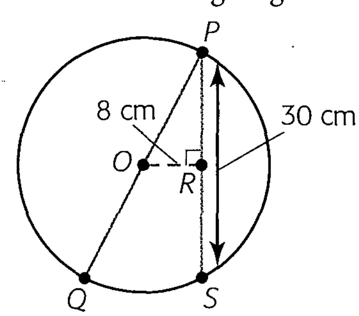
2. A sand timer can be set into a circle as shown. The heights of the top and bottom sections are equal. Are the widths of the top and bottom the same? Explain how you know.



Practising

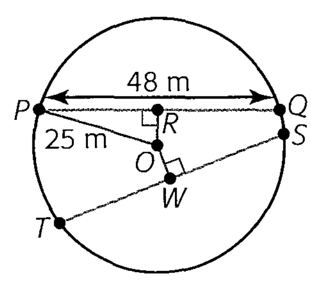
3. Calculate the missing lengths to the nearest unit.

a)



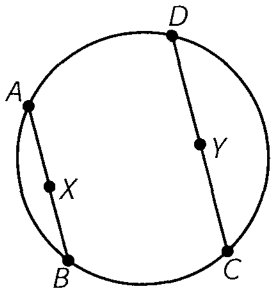
$PQ = ?$

b)

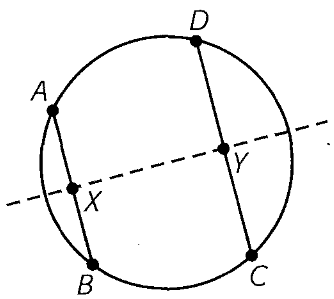


$OW = 7$ m, $ST = ?$

Zachary's Solution: Trying with parallel chords



I wondered if it was possible to locate the centre of a circle if the chords were parallel.
I drew a circle with two parallel chords, *AB* and *CD*.
I constructed the midpoints, *X* and *Y*, of each chord.



I constructed a perpendicular line through *X*. I noticed that the perpendicular bisector of *AB* went through *Y*. Then I constructed a perpendicular line through *Y*. The perpendicular bisector of *CD* went through *X*.

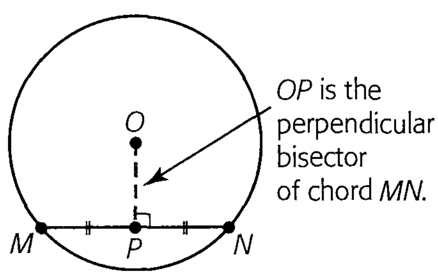
If two chords are parallel, you cannot locate the centre of a circle directly using only their perpendicular bisectors.

Since both perpendicular bisectors are the same line, I could not determine the location of the centre.
I know the centre is on line segment *XY*, but I don't know where.

In Summary

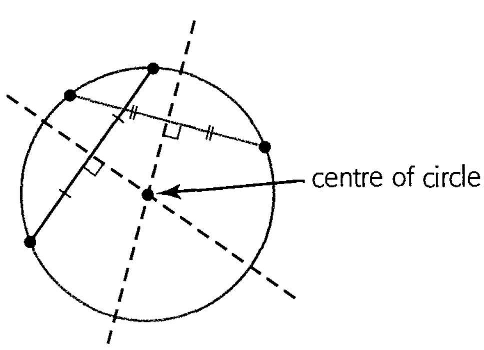
Key Idea

- A line that passes through the centre of a circle and the midpoint of a chord is perpendicular to the chord. Another way of saying this is that a line that is perpendicular to a chord and also passes through the centre of the circle bisects the chord.



Need to Know

- The perpendicular bisector of a chord passes through the centre of a circle.
- The centre of a circle is located at the intersection of the perpendicular bisectors of two non-parallel chords.

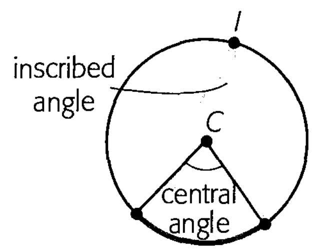


In Summary

Key Ideas

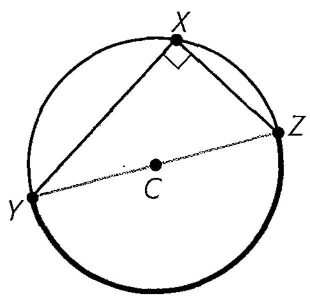
- An inscribed angle is equal to half the measure of the central angle subtended by the same arc.

$$\angle I = \frac{1}{2}\angle C \text{ or } \angle C = 2\angle I$$



Need to Know

- An inscribed angle subtended by a semicircle measures 90°.

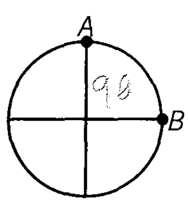


Since YZ is a diameter, arc YZ is a semicircle. $\angle YCZ$ is a central angle equal measuring 180°. $\angle X$ is subtended by arc YZ, so $\angle X = 90°$.

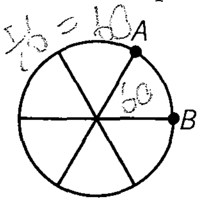
Checking

1. Determine the measure of each central angle subtended by minor arc AB. The radii divide each circle into equal parts.

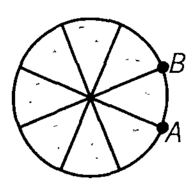
a)



b)

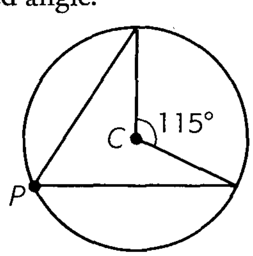


c)



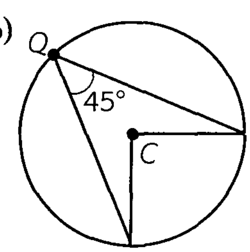
2. For each circle with centre C, determine the measure of the red angle.

a)



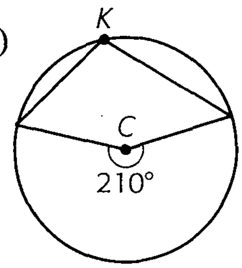
$\angle P = ?$

b)



$\angle C = ?$

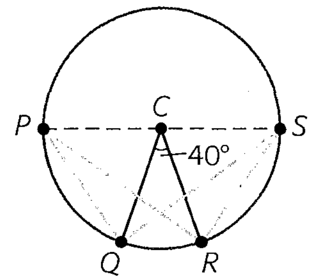
c)



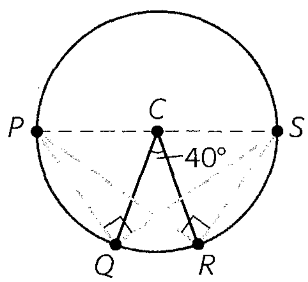
$\angle K = ?$

EXAMPLE 2 Determining missing angles

A magician is designing a logo for his business. His logo is drawn in a circle centred at *C*. What are the measures of $\angle QPR$, $\angle PQS$, $\angle PRS$, and $\angle QSR$ in the logo?



Zachary's Solution



I copied the logo.
I knew that *PS* is a diameter because it goes through the centre of the circle, so arc *PS* is a semicircle. This means that $\angle PQS$ and $\angle PRS$ are inscribed angles subtended by a semicircle, so they are each 90°.

$\angle PQS = \angle PRS = 90°$

$\angle QPR = \left(\frac{1}{2}\right)\angle QCR$

$= \frac{1}{2} \times 40°$

$= 20°$

$\angle QPR = \angle QSR = 20°$

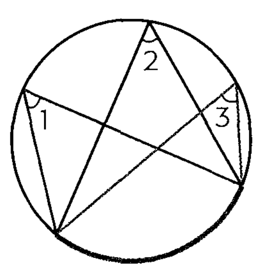
I noticed that $\angle QPR$ and $\angle QSR$ are inscribed angles subtended by minor arc *QR*. This minor arc also subtends central $\angle QCR$. So $\angle QPR$ and $\angle QSR$ are half of $\angle QCR$.

In the logo, $\angle QPR = \angle QSR = 20°$ and $\angle PQS = \angle PRS = 90°$.

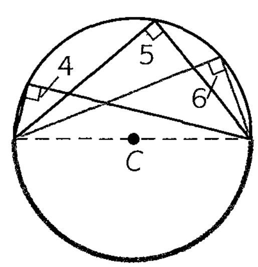
In Summary

Key Idea

- It is possible to have many inscribed angles subtended by the same arc. Angles 1, 2, and 3 have the same measure. If the arc is a semicircle, the inscribed angles are 90°.



$\angle 1 = \angle 2 = \angle 3$



$\angle 4 = \angle 5 = \angle 6 = 90°$